$\int u\,dv = uv - \int v\,du$

MA114 Summer II 2018
Worksheet 1a

Solutions

1. Compute the following integrals by using integration by parts.

a) $\int x^2 \sin(x)\,dx$,

$u = x^2 \quad dv = \sin(x)\,dx$
$du = 2x\,dx \quad v = -\cos(x)$

$= -x^2\cos(x) - \int -2x\cos(x)\,dx$

$= -x^2\cos(x) + 2\int x\cos(x)\,dx$ $\quad u = 2x \quad dv = \cos(x)\,dx$
$du = 2dx \quad v = \sin(x)$

$= -x^2\cos(x) + 2x\sin(x) - \int 2\sin(x)\,dx$

$= \boxed{-x^2\cos(x) + 2x\sin(x) + 2\cos(x) + C}$

b) $\int (2x+1)e^x\,dx$,

$u = 2x+1 \quad dv = e^x dx$
$du = 2dx \quad v = e^x$

$= (2x+1)e^x - \int 2e^x dx$

$= \boxed{(2x+1)e^x - 2e^x + C}$

or

$\boxed{(2x-1)e^x + C}$

c) $\int x^4 \ln(x)\,dx$,

$u = \ln(x) \quad dv = x^4 dx$
$du = \frac{1}{x}dx \quad v = \frac{1}{5}x^5$

$= \frac{1}{5}x^5\ln(x) - \int \frac{1}{5}x^5 \cdot \frac{1}{x}dx$

$= \frac{1}{5}x^5\ln(x) - \frac{1}{5}\int x^4 dx$

$= \boxed{\frac{1}{5}x^5\ln(x) - \frac{1}{25}x^5 + C}$

d) $\int x\ln(x+1)\,dx$,

$u = \ln(x+1) \quad dv = x\,dx$
$du = \frac{1}{x+1}dx \quad v = \frac{1}{2}x^2$

$= \frac{1}{2}x^2\ln(x+1) - \int \frac{1}{2}\frac{x^2}{x+1}dx$

Substitute $t = x+1 \quad x = t-1$
$dt = dx$

$= \frac{1}{2}x^2\ln(x+1) - \int \frac{1}{2}\frac{(t-1)^2}{t}dt$

$= \frac{1}{2}x^2\ln(x+1) - \int \frac{1}{2}\frac{t^2-2t+1}{t}dt$

$= \frac{1}{2}x^2\ln(x+1) - \frac{1}{2}\int t - 2 + \frac{1}{t}dt$

$= \frac{1}{2}x^2\ln(x+1) - \frac{1}{4}t^2 + t - \frac{1}{2}\ln|t| + C$

$= \boxed{\frac{1}{2}x^2\ln(x+1) - \frac{1}{4}(x+1)^2 + (x+1) - \frac{1}{2}\ln|x+1| + C}$

(alt. solution on back)

$\int x \ln(x+1)\,dx$: Substitute $t = x+1$ $\quad x = t-1$

$dt = dx$

$= \int (t-1)\ln(t)\,dt$

$= \int t\ln(t)\,dt - \int \ln(t)\,dt$

$\int t\ln(t)\,dt =$ $\quad u = \ln(t)$ $\quad dv = t\,dt$

$du = \frac{1}{t}dt$ $\quad v = \frac{1}{2}t^2$

$= \frac{1}{2}t^2\ln(t) - \int \frac{1}{2}t\,dt$

$= \frac{1}{2}t^2\ln(t) - \frac{1}{4}t^2$

$\int \ln(t)\,dt$ $\quad u = \ln(t)$ $\quad dv = dt$

$du = \frac{1}{t}dt$ $\quad v = t$

$= t\ln(t) - \int 1\,dt$

$= t\ln(t) - t$

So: $\int x\ln(x+1)\,dx = \frac{1}{2}t^2\ln(t) - \frac{1}{4}t^2 - t\ln(t) + t + C$

$= \boxed{\frac{1}{2}(x+1)^2\ln(x+1) - \frac{1}{4}(x+1)^2 - (x+1)\ln(x+1) + (x+1) + C}$